Exam time: 3 hours

Instructions:
1. For writing your answers use both sides of the paper in the answer booklet.

2. Additional sheets taken, if any, should be properly attached to the main answer booklet.

3. **(5 points) Please write your name on every page of this booklet and every additional sheet taken.**
4. Maximum time is 3 hours and Maximum Possible Score is 100

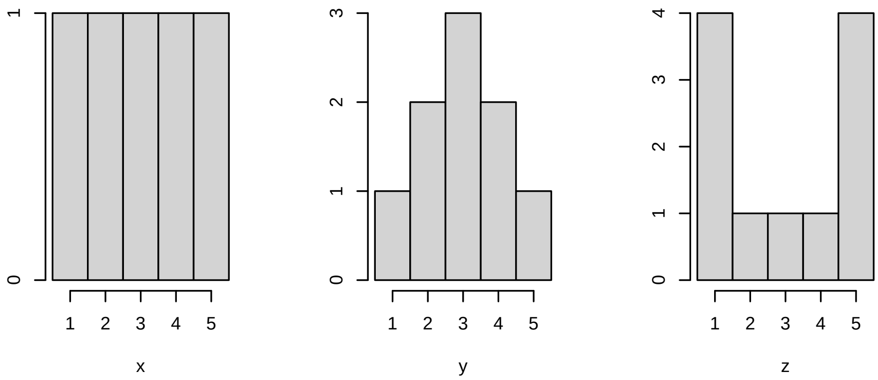
Score

Q.No.	**Alloted Score**	**Score**
1.	(15 points)	
2.	(15 points)	
3.	(15 points)	
4.	(20 points)	
5.	(20 points)	
6.	(15 points)	
7.	(10 points)	
Total	110	

Number of Extra sheets attached to the answer script: ____________

1. In each of the following below, please circle the correct choice. No justification is required.

(A): Consider the three histograms given below of datasets `x`, `y` and `z`



The ordering of the dataset from the smallest to biggest standard deviations is given by:

(i)(`x`, `y`, `z`) (ii)(`x`, `z`, `y`) (iii)(`y`, `x`, `z`) (iv)(`y`, `z`, `x`) (v)(`z`, `y`, `x`) (vi)(`z`, `x`, `y`)

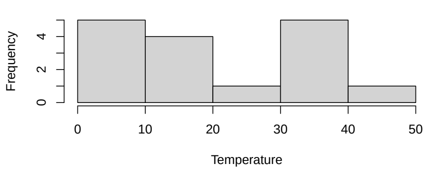
(B): Suppose we are given $X_1, X_2, \dots X_n$ i.i.d sample from a population X. Suppose an estimate for θ is given by $\hat{\theta} = g(X_1, X_2, \dots X_n)$ and $\hat{\theta}_i^\star = g_{(i)}(X_1, X_2 \dots, X_{i-1}, X_{i+1}, \dots X_n)$ for $i = 1, \dots n$. Then the jackknife bias is given by:

(i) $(n-1)\left(\frac{1}{n}\sum_{i=1}^{n}\hat{\theta}_i^\star - \hat{\theta}\right)$ (ii) $\left(\frac{1}{n}\sum_{i=1}^{n}\hat{\theta}_i^\star - \hat{\theta}\right)$ (iii) $\min\{\mid \hat{\theta}_i^\star - \hat{\theta} \mid\}$

(C) Fill in (I),(II),(III), (IV),(V)

```
Temperature = c(6,8,15,19,(I),32,37,35,(II),32,31,15,19,8,9,0)
(III)(Temperature, xlab="(IV)", main = "", ylab="(V)")
```

so that the above `R` produces the plot below.



2. (15 points) At the ISI co-rec basketball league in the 10 games played team *Unit-disc* scored:

$$59, 62, 59, 74, 70, 61, 62, 66, 62, 75$$

Assume that the number of points scored by *Unit-disc* is Normally distributed.

(a) Compute a 95% confidence interval for the mean, μ.

(b) We want to test the null hypothesis that $\mu = 63$ versus the alternative hypothesis that $\mu \neq 63$. Decide and execute a test that can check if there is enough evidence whether one can reject the null hypothesis at 5% level of significance.

3. (15 points) Suppose we toss a coin repeatedly and observe that the first head appears in the 5-th trial. Find the maximum likelihood estimate for the probability of heads when one tosses the coin.

4 (20 points)The following `R` code simulates a random variable `X`

```
> L = 10
> i = 0
> U = runif(1, min=0, max =1)
> Y = -log(U)/L
> Sum = Y
> while (Sum<1) {
+     U = runif(1, min=0, max =1)
+     Y = -log(U)/L
+     Sum = Sum +Y
+     i = i + 1
+ }
> X = i
```

Find the distribution of X (*Other than p.d.f. or p.m.f. of standard distribution functions please provide adequate justification of any result that you are using*).

5. (20 points) Shyamala is interested in studying average years of schooling in India. They hypothesized that the mean years of school for people 18 years old or above is higher than 8.5 years. They drew a sample of 300000 from 2011 census and find out that the mean number of years of schooling for the sample is 8.515, with a SD of 4.5. We wish to test

$$H_0 : \mu \leq 8.5 \text{ versus } H_A : \mu > 8.5$$

   (a) Compute the T-statistic.
   (b) Find the p-value.
   (c) Describe the meaning of p-value and what inference can you draw from it if your level of significance is 0.005

6. (15 points) The student body at an undergraduate university is 20% Masters, 24% third years, 26% Second year, and 30% first year students. Suppose a researcher takes a sample of 50 such students. Within the sample there are 13 Masters, 16 Third years, 10 Second years, and 11 First years. The researcher claims that his sampling procedure should have produced independent selections from the student body, with each student equally likely to be selected. Is this a plausible claim given the observed results?

7. (10 points) X and Y are two independent samples from populations with distributions F and G respectively. We want to test $F = G$. Suppose,

$$X : 53, 38, 69, 57, 46, 39, 73, 48, 73, 74, 60, 78$$

and

$$Y : 44, 40, 61, 52, 32, 44, 70, 41, 67, 72, 53, 72,$$

Compute the Mann-Whitney U test statistic (discussed in class).

Statistical Tables for Reference if Needed.

Table 1 : in (i, j)-th entry provides $\mathbb{P}(t_i \leq j)$, with $t_i \sim$ t-distribution with i degrees of freedom.

	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
2	0.570	0.604	0.636	0.667	0.695	0.722	0.746	0.768	0.789	0.807	0.823
3	0.573	0.608	0.642	0.674	0.705	0.733	0.759	0.783	0.804	0.824	0.842
4	0.574	0.610	0.645	0.678	0.710	0.739	0.766	0.790	0.813	0.833	0.852
5	0.575	0.612	0.647	0.681	0.713	0.742	0.770	0.795	0.818	0.839	0.858
6	0.576	0.613	0.648	0.683	0.715	0.745	0.773	0.799	0.822	0.843	0.862
7	0.576	0.614	0.649	0.684	0.716	0.747	0.775	0.801	0.825	0.846	0.865
8	0.577	0.614	0.650	0.685	0.717	0.748	0.777	0.803	0.827	0.848	0.868
9	0.577	0.615	0.651	0.685	0.718	0.749	0.778	0.804	0.828	0.850	0.870

Table 2 : in (i, j)-th entry provides $\mathbb{P}(t_i \leq j)$, with $t_i \sim$ t-distribution with i degrees of freedom.

	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
1	0.750	0.765	0.779	0.791	0.803	0.813	0.822	0.831	0.839	0.846	0.852
2	0.789	0.807	0.823	0.838	0.852	0.864	0.875	0.884	0.893	0.901	0.908
3	0.804	0.824	0.842	0.858	0.872	0.885	0.896	0.906	0.915	0.923	0.930
4	0.813	0.833	0.852	0.868	0.883	0.896	0.908	0.918	0.927	0.935	0.942
5	0.818	0.839	0.858	0.875	0.890	0.903	0.915	0.925	0.934	0.942	0.949
6	0.822	0.843	0.862	0.879	0.894	0.908	0.920	0.930	0.939	0.947	0.954
7	0.825	0.846	0.865	0.883	0.898	0.911	0.923	0.934	0.943	0.950	0.957
8	0.827	0.848	0.868	0.885	0.900	0.914	0.926	0.936	0.945	0.953	0.960
9	0.828	0.850	0.870	0.887	0.902	0.916	0.928	0.938	0.947	0.955	0.962
10	0.830	0.851	0.871	0.889	0.904	0.918	0.930	0.940	0.949	0.957	0.963

Table 3 : in (i, j)-th entry provides $\mathbb{P}(t_i \leq j)$, with $t_i \sim$ t-distribution with i degrees of freedom.

	2.20	2.22	2.24	2.26	2.28	2.30	2.32	2.34	2.36	2.38	2.40
2	0.921	0.922	0.923	0.924	0.925	0.926	0.927	0.928	0.929	0.930	0.931
3	0.942	0.943	0.945	0.946	0.947	0.948	0.948	0.949	0.950	0.951	0.952
4	0.954	0.955	0.956	0.957	0.958	0.959	0.959	0.960	0.961	0.962	0.963
5	0.960	0.961	0.962	0.963	0.964	0.965	0.966	0.967	0.968	0.968	0.969
6	0.965	0.966	0.967	0.968	0.969	0.969	0.970	0.971	0.972	0.973	0.973
7	0.968	0.969	0.970	0.971	0.972	0.973	0.973	0.974	0.975	0.976	0.976
8	0.971	0.971	0.972	0.973	0.974	0.975	0.976	0.976	0.977	0.978	0.978
9	0.972	0.973	0.974	0.975	0.976	0.977	0.977	0.978	0.979	0.979	0.980

Table 4: in (i, j)-th entry provides $\mathbb{P}(Z \leq i + j)$, with $Z \sim \text{Normal}(0, 1)$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	0.5398
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	0.5793
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	0.6179
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	0.6554
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	0.6915
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	0.7257
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	0.7580
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	0.7881
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	0.8159
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	0.8413
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	0.8643
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	0.8849
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	0.9032
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	0.9192
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	0.9332
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	0.9452
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	0.9554
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	0.9641
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	0.9713
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	0.9772
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0.9821
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0.9861
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0.9893
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0.9918
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0.9938
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0.9953
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0.9965
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0.9987
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998

April 30th, 2026 **Name (Please Print)** ______________________________

Statistical Tables for Reference if Needed.

Table 5: in (i, j)-th entry provides $\mathbb{P}(\chi_i^2 \leq j)$, with $\chi_i^2 \sim$ Chi-square with i degrees of freedom.

	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
2	0.181	0.221	0.259	0.295	0.330	0.362	0.393	0.423	0.451	0.478	0.503
3	0.060	0.081	0.104	0.127	0.151	0.175	0.199	0.223	0.247	0.271	0.294
4	0.018	0.026	0.037	0.049	0.062	0.075	0.090	0.106	0.122	0.139	0.156
5	0.005	0.008	0.012	0.017	0.023	0.030	0.037	0.046	0.055	0.065	0.076
6	0.001	0.002	0.004	0.006	0.008	0.011	0.014	0.018	0.023	0.028	0.034
7	0.000	0.001	0.001	0.002	0.003	0.004	0.005	0.007	0.009	0.012	0.014
8	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.002	0.003	0.004	0.006
9	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.002	0.002

Table 6: in (i, j)-th entry provides $\mathbb{P}(\chi_i^2 \leq j)$, with $\chi_i^2 \sim$ Chi-square with i degrees of freedom.

	2.50	2.52	2.54	2.56	2.58	2.60	2.62	2.64	2.66	2.68	2.70
1	0.886	0.888	0.889	0.890	0.892	0.893	0.894	0.896	0.897	0.898	0.900
2	0.713	0.716	0.719	0.722	0.725	0.727	0.730	0.733	0.736	0.738	0.741
3	0.525	0.528	0.532	0.535	0.539	0.543	0.546	0.549	0.553	0.556	0.560
4	0.355	0.359	0.363	0.366	0.370	0.373	0.377	0.380	0.384	0.387	0.391
5	0.224	0.227	0.230	0.233	0.236	0.239	0.242	0.245	0.248	0.251	0.254
6	0.132	0.134	0.136	0.138	0.141	0.143	0.145	0.148	0.150	0.152	0.155
7	0.073	0.074	0.076	0.077	0.079	0.081	0.082	0.084	0.085	0.087	0.089
8	0.038	0.039	0.040	0.041	0.042	0.043	0.044	0.045	0.046	0.047	0.048
9	0.019	0.020	0.020	0.021	0.021	0.022	0.023	0.023	0.024	0.024	0.025
10	0.009	0.009	0.010	0.010	0.010	0.011	0.011	0.011	0.012	0.012	0.012

Table 7: in (i, j)-th entry provides $\mathbb{P}(\chi_i^2 \leq j)$, with $\chi_i^2 \sim$ Chi-square with i degrees of freedom.

	2.50	2.52	2.54	2.56	2.58	2.60	2.62	2.64	2.66	2.68	2.70
2	0.713	0.716	0.719	0.722	0.725	0.727	0.730	0.733	0.736	0.738	0.741
3	0.525	0.528	0.532	0.535	0.539	0.543	0.546	0.549	0.553	0.556	0.560
4	0.355	0.359	0.363	0.366	0.370	0.373	0.377	0.380	0.384	0.387	0.391
5	0.224	0.227	0.230	0.233	0.236	0.239	0.242	0.245	0.248	0.251	0.254
6	0.132	0.134	0.136	0.138	0.141	0.143	0.145	0.148	0.150	0.152	0.155
7	0.073	0.074	0.076	0.077	0.079	0.081	0.082	0.084	0.085	0.087	0.089
8	0.038	0.039	0.040	0.041	0.042	0.043	0.044	0.045	0.046	0.047	0.048
9	0.019	0.020	0.020	0.021	0.021	0.022	0.023	0.023	0.024	0.024	0.025